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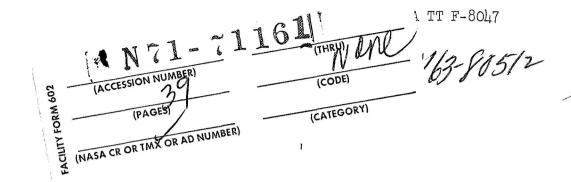
# THE PROBLEM OF POSITION OBSERVATIONS OF ARTIFICIAL EARTH SATELLITES AND THE DETERMINATION OF THE GEOGRAPHIC COORDINATES OF SUBSATELLITE POINTS

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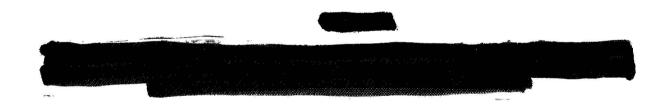
THE PROBLEM OF POSITION OBSERVATIONS OF ARTIFICIAL EARTH SATELLITES

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POINTS

The first part of the article discusses the importance of
artificial earth-satellite position observations in general, and
presents a review of the scientific and practical applications of
these observations in particular in the fields of geodesy, cartography
and practical astronomy. Theorems and formulas related to the theory
of artificial earth-satellite orbits, which will be applied in the
practical work presented in the second part, are also examined.

The second part examines a numerical example of geographic
position determination, and distance from the surface of the earth
of artificial earth-satellite 1958$\delta$2 (Sputnik III) for specific
moments, according to preliminary determination of the missing elements
of its orbit and their changes in time, on the basis of visual obser-
vations in the period from the second half of May through the first
half of June 1958.

PART I

A. Introduction

From autumn 1957 to the present time (end of April 1960),
scientists of the Soviet Union and the United States have put 21
artificial earth satellites and three artificial planets into orbits
around the earth. These include: 3 Sputniks, 5 Explorers, 3 Vanguards,
7 Discoverers, 1 Atlas, 1 Transit, 3 Lunniks, and 2 Pioneers. Taking
into account the multiple elements of the objects listed, e.g., in
the case of Sputnik III we have: 1958$\delta$1 - the rocket carrier, 1958$\delta$2 -
the nose cone and container, 1958$\delta$3 - the nose cover, 1958$\delta$4 and 5 -
the side covers, which have penetrated outer space, several dozen man-
made celestial bodies now exist. The inclination of the planes of

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these objects to the equatorial plane of the earth ranges from about 33° in the case of the Vanguards and some of the Explorers, about 50° in the case of the remaining Explorers, about 65° in regard to the Soviet sputniks, and up to 90° in the case of the American Discoverers. When one considers that a significant number of the satellites put into orbit are still there and will rotate around our planet several decades, or several hundred years, or theoretically, for ever, and that the number of automatic space vehicles will increase with time, one can easily see that the first stage in the peaceful mastery of outer space is being realized before our eyes. The next stage--manned penetration of interplanetary space and a landing on the Moon or another planet-- is also a matter for the immediate years ahead and will undoubtedly be accomplished in this generation.

The question, however, is whether the launching of artificial earth and sun satellites is a shining step forward solely in the field of rocket technology and astronautics? A multitude of theoretical works and the processing of observations and communications which emanate from scientific centers, as well as the ever increasing field of scientific disciplines contradict such an assumption.

From the nature of things, the use of satellites in scientific investigations has brought about general progress in sciences dealing with the earth and its immediate environment (1): in geophysics, with special attention to the physics of the upper atmospheric layers (pressure, density, temperature, chemical composition, etc.); in the sciences of the electrostatic and electromagnetic fields of the earth, and in meteorology. Observations conducted with the aid of satellites have also provided man new scientific perspectives in understanding the problems of outer space, such as meteoritics, the study of the structure of interstellar gaseous matter, cosmic radiation, solar radiation, and the short-wave region of the solar spectrum.

New roads have been opened to astronomy, due to the possibility of carrying out various observations without interference caused by the earth's atmosphere, which can alter photography of bodies in the solar system and metagalaxies.

Biology and medicine are today concerned with the question of the effects of space flight on a human being (acceleration, noise and vibration, weightlessness), and at the same time are making equipment which will provide living conditions for the crew, i.e., air regeneration, temperature regulation, water supply, fixation, etc.

In the experimental stage are the problems of world-wide telecommunications and television through the use of three stationary satellites in circumequatorial orbit at a distance of about 36,000 km from the earth. Projects calling for interplanetary bases for military purposes are also known.

Finally, we shall speak about the use of satellites in the fields of higher geodesy, cartography, geodetic and navigational astronomy.

The overwhelming majority of scientific and investigatory material in the aforementioned fields of science obtained through the use of artificial micromoons and microplanets, is received via the installation of a variety of automatic equipment in a container which transmits the information to the earth by radio.

Especially important, both for science as well as for practical purposes, are satellite position observations. These are: visual, photographic, and radar. Those kinds of observations and the materials obtained through them are, naturally, chiefly in the sphere of interest of geodesy and position astronomy.

If we overlook the above-mentioned applications, which may be somewhat overenthusiastic, though still not too premature for study in geodetic measuring and astronomy, that await us in the not-distant future (the problem of determining position and time, the determination of the figure, surface measurements and distances...on the moon or near planet) -- present knowledge and geodetic astronomical practice has been enriched, thanks to the sputnik flights, with new research and practical perspectives. Here are some of the more important ones.

The earth satellite has executed its revolving flight within the gravitational field of the earth. Its orbit, according to the laws of celestial mechanics, must maintain a constant orientation in space and constant shape and dimensions. Deformation of the orbit and changes in the time of the elements of orientation are caused (in the case of not too distant sputniks) chiefly by two agents of perturbation: atmospheric resistance and the oblateness of the earth's elipsoid. Given a sufficient knowledge of the first of these agents, and, based on the quantitative effect of the influence of the equatorial bulge on change of orbital elements obtained from position observations, we can derive the value for the oblateness of the earth. (The oblateness value obtained in this way by Soviet scientists is $1/297.5\pm0.5$; that computed by USA scientists on the other hand is $1/298.2$)

In an analogous way, based on the observational results of small deformations and orbital changes as a result of the probable existence of the three axes of symmetry of the earth (two in the equatorial plane) of its pear or barrel shape, we can obtain data relative to the figure of the geoid and the distribution of masses within it.

Artificial earth satellites also provide the possibility of designing a new method of establishing relative ellipsoids by determining the positions of their geometric centers relative to the center of the earth's force.

A new road has been opened to the practical solution of problems of intercontinental communications. The launching of the first navigational satellite (Transit 1) brought the field of naval and air navigation, as well as related methods of astronavigation and the determination of approximate geographic positions to the stage of practical realization.

New possibilities now exist in the problem of time determination through the help of observations of the passage of an artificial celestial body free from the influence of the resistance of the terrestrial atmosphere.

Finally, in the field of mapping isolated and little-known regions, the techniques of radar measurements have found remarkable application. In this way the geographic positions of a series of islands in the Pacific have been corrected on the maps.

Position observations have special importance in the experimental checking of certain effects of the theory of relatively, since one year of observations of the flight of an artificial satellite has the same value as observing Mercury for a period of 40 years; the relativistic effect of shifting of the perigee of the artificial satellite is about the same ratio greater than the movement of Mercury's perihelium.

As we see from this rapid survey, the importance of satellite position observations for a whole range of problems in geodesy and astronomy is undoubted.

In the introduction of this article we have restricted ourselves solely to indicating some of the more important problems, not going too deeply into the theory of any specific problem, nor presenting the actual state of knowledge achieved in the most recent investigations. The aim of this work, therefore, after noting the importance of position observations to satisfy geodetic astronomical demands, is to touch the problem of the relationship between orbit elements and parameters measured in the course of observations and certain perturbation effects, and then in the second part to treat a numerical work problem analogous or similar to those fundamental in the processing of observational material for various purposes.

B. Artificial Satellite Observations

Being aware of the great general importance and, in some questions, the urgency of optical satellite observations, the continental powers, the Soviet Union and the United States, had, even before the launching of the first Sputnik and Explorer, prepared a network of stations to provide a constant observation service. The number of visual satellite observation stations at the present time is about 500. Of these, about 100 are in the USSR, 150 in the USA, 80 in Japan, and so on. In populous countries on all continents the number ranges from several to about 20. These stations, linked by telegraph through disposition centers (KOSMOS in the USSR, the Moonwatch organization of the Smithsonian Institution in the USA), report in their observations the topocentric equatorial or horizontal coordinates and corresponding times for each passage. The accuracy of these observations, carried out with small telescopes (small magnification, large field of view), theodolites, or binoculars, is of the order of $0.1^{\circ} - 1^{\circ}$ (coordinates) and $0.1^{S} - 1^{S}$ (time). Mass visual observations provide supplementary material for the determination of approximate satellite orbit elements and for the computation of ephemerides for specific stations.

Observational techniues differ. Where there is a great number of observational stions and numerous observers, the optical barrier method or the meridian passages and first vertical method is applied; where there is a small number of observers the sky patrol method is used. Time may be recorded by the visual-sound method or with the use of a chronograph. Visual observations are indispensable in the case of objects of little brightness, and are especially important in the initial and final stages of satellite life, when its orbit has not yet been sufficiently determined or when its elements are changing.

Important accurate results are achieved in observations using photographic cameras, $0.01^{\circ}$ - $0.1^{\circ}$ and $0.01^{s}$ - $0.1^{s}$ respectively. Moreover, these observations are quite economical, since it is possible to read several satellite positions from a single negative. A shortcoming of the photographic method is its limited applicability in the case of objects characterized by a small degree of brightness.

Some 12 special stations, equipped with large Baker-Nunn cameras of precise construction, capable of accuracies to $0.001^{\circ}$ and $0.001^{s}$, are now set up in the circumequatorial zone.

The regular Polish Satellite-Observation Service, established in the spring of 1958, in connection with the IGY, has 10 visual observation stations. Two of these can also make photographic observations and one can perform radio monitoring (13). In the beginning of 1960, university and polytechnic stations in Warsaw, Krakow, Poznan, and Wroclaw, as well as the PTMA station in Gdansk, the Planetarium in Chrzov, and the station in Zegrz, announced their readiness to conduct continual observations. At least three of the above-mentioned stations will be able to carry on photographic observations, one will continue radio observations.

The Polish Satellite Service, coordinated by the Committee of International Geophysical Cooperation, works together with the disposition center in the Soviet Union (KOSMOS). The State Hydrological and Meteorological Institute cooperates closely with the national service. It has made its communications equipment available for liaison between the Kosmos bureau, the IGY Committee, and the individual stations.

The results of Polish satellite observations have been published in Soviet bulletins containing collected works (3). Since the beginning of 1960, however, they have appeared in the "Bulletin of Polish Satellite Observations" (4) together with other related information on sputniks.

C.  Practical Computational Problems

The following two basic computational problems occur in cases demanding a strict numerical solution, as well as in regard to approximation problems, depending upon circumstances:

1. On the basis of observational materials (topocentric equatorial or horizontal satellite coordinates and the moments of passage; the geographic coordinates of the station), compute the orbit elements and changes in time.

2. Given the satellite orbit elements, compute the geographic position of subsatellite points and the height of the satellite above the surface of the earth for definite moments of time.

In the second part of this work we present an example of the solution of a specific tractical problem in which two conditions have to be satisfied. The most general development of the problems leads to the determination of the geographic coordinates $\varphi_s$ and $\lambda_s$ and the heights $H_s$ of satellite 1958$\delta$ 2 above the earth's surface for given moments of time of a definite epoch. The difficulty in the solution of the problem lies in the lack of orbit elements linking satellite position with time, and in only having approximate knowledge of the remaining elements.

The above-mentioned problem is the geodetic astronomical supplement of the scientific processing of the radio observations of the Zegrze station, having as its aim, among other things, the verification of S. Manczarski's theory on the propagationof radio waves in the ionosphere (5). Both the ionospheric and geodetic astronomical part of the problem were published in the form of abstracts at the 3rd Conference of Representatives of The Eurasian Region IGY, in February 1959 (9).

In this work only the main line of computations, together with pertinent formulas and a discussion of them, is given.

Now we shall treat certain theorems and formulas, as well as theoretical considerations dealing with the motion of artificial earth satellites. On the basis of these, a discussion of the problem and the choice of solution follows.

D. Elliptical Satellite Orbit

The motion of the satellite having mass $m_1$ around the earth with mass $m_2$ occurs in the gravitational field

$$K = \frac{fm_1m_2}{r^2} , \tag{1}$$

where f is the constant universal gravity, r is the radius to the current satellite position (its distance from the center of the earth). The motion of the satellite occurs according to the equations of motion:

$$\frac{d^2 r}{dT^2} - r \left(\frac{dv}{dT}\right)^2 = \frac{fm_2}{r^2} \qquad (2a)$$

and

$$\frac{d(r^2 v)}{dT} = 0, \qquad (2b)$$

Angle v, which with the value r, characterize the position of the satellites in the plane of the orbit is a plane angle that can be identified with the so-called true anomaly, measured from perigee P (Diagram 1) according to the direction of satellite motion.

Integrating equation 2b leads to Kepler's second law, which we express with the formula:

$$r^2 \frac{dv}{dT} = 2 \frac{d\dot{\varphi}}{dT} = C, \qquad (3)$$

where C is the so-called constant of the law of areas, while $\frac{d\dot{\varphi}}{dT} = \frac{r^2}{2} \frac{dv}{dT}$ is the field of areas described by radius r in a unit of time. This is the so-called sector velocity, during which the velocity of the satellite in its orbit is

$$V^2 = \left(\frac{dr}{dT}\right)^2 + r^2 \left(\frac{dv}{dT}\right)^2 \qquad (4)$$

Between the velocity vector V and the r vector the fllg. relationship is set up:

$$\frac{dr}{dT} = V \cos \gamma, \qquad (5)$$

where $\gamma$ (Diagram 1) is the angle contained between the direction of satellite motion and the direction at which it is visible from the earth. The relationship (5) expresses the radial velocity, while the expression for the angular velocity has the form:

$$r \frac{dv}{dT} = V \sin \gamma \qquad (6)$$

On the basis of relationships (3) and (4), we can write:

$$C = V r \sin \gamma = V_p r_p; \qquad (7)$$

where the symbol p refers to the point of the satellite's closest approach to the surface of the earth, called the perigee (Diagram 1). We have therefore

$$\frac{m_1}{2} V^2 - \frac{fm_1 m_2}{r} = \text{total energy} \qquad (8)$$

$$\underset{\text{kinetic}}{\text{energy}} + \underset{\text{potential}}{\text{energy}}$$

On the strength of relationships (4) and (8), after elimination of time T, and with equation (3), and integrating we get the equation:

$$r = \frac{a(1 - e^2)}{1 + e \cos v} \quad , \tag{9}$$

expressing Kepler's first law.

If $\hat{\varphi} = \pi.a.b.$ is the total area circumscribed by the orbit, while P is the period of satellite revolution, then using formula (3), we arrive at the relationship

$$c = 2 \frac{\pi ab}{P} = \frac{2\pi}{P} a^2 (1 - e^2)^{1/2} \quad . \tag{10}$$

And since at the same time

$$c^2 = fm_2 a(1 - e^2), \tag{11}$$

taking equations (10) and (11) we get the relationship:

$$\frac{a^3}{p^2} = \frac{fm_2}{4\pi^2} = \text{const.}, \tag{12}$$

which bears the name of Kepler's third law.

Formulas (3), (9), and (12), characterizing the law of satellite motion along its elliptical orbit, may be supplemented with the words: an artificial earth satellite moves in the same plane that passes through the center of the earth, while the position of that plane does not change in relation to the stars; the satellite orbit is an ellipse, one focus of which is in the center of the earth; the revolution period of a satellite is related to its distance from the center of the earth. We assume here that the earth is a homogeneous sphere.

The position of a satellite in its orbit at a given moment is defined by the formulas, which result from the following considerations. From equations (3) and (9) (Kepler's first and second laws) we get:

$$T - T_p = \frac{1}{C} \int_0^v r^2 dv = \frac{a^2(1 - e^2)^2}{C} \int_0^v \frac{dv}{(1 + e \cos v)^2} \tag{13}$$

Introducing a known relationship from celestial mechanics

$$\text{tg} \frac{v}{2} = \sqrt{\frac{1 + e}{1 - e}} \; \text{tg} \frac{E}{2} \; ,$$

where E designates the eccentric anomoly, after substitution and integration, we get on the strength of (13) and (11):

$$T - T_p = \frac{ab}{C} (E - e \sin E) = \frac{P}{2\pi} (E - e \sin E), \tag{14}$$

whence we get the so-called Kepler's equation:

$$E - e \sin E = \frac{2\pi}{\tau}(T - T_p) = M \text{ (so-called mean anomaly)}. \tag{15}$$

In the case where it is necessary to determine the position of the satellite in its orbit at a definite moment T, the application of Kepler's equation (15) meets difficulties, since it is an equation applicable in regard to the eccentric anomaly E. We use in this case a form of this formula expanded in series with the aid of the Bessel function, i.e.,

$$E = M + e\left(1 - \frac{e^2}{8}\right)\sin M + \frac{e^2}{2}\sin 2M + \frac{3}{8}e^3\sin 3M + \ldots \tag{16}$$

In a similar way, the expression for the true anomaly v and the mean anomaly M are expanded in series:

$$v = M + 2e\left(1 - \frac{e^2}{8}\right)\sin M + \frac{5}{4}e^2\sin 2M + \frac{13}{12}e^3\sin 3M + \ldots \tag{17}$$

$$M = v + 2e\left(1 - \frac{5e^2}{4}\right)\sin v + \frac{13}{4}e^2\sin 2v + \frac{47}{6}e^3\sin 3v + \ldots \tag{18}$$

In its turn, the relationship of the value of the moving radius (the distance of the satellite from the center of the earth) to the major semiaxis:

$$\frac{r}{a} = \frac{1 - e^2}{1 + e\cos v} = 1 - e\cos E$$

we can put in series:

$$\frac{r}{a} = 1 + \frac{e^2}{2} - e\left(1 - \frac{3e^2}{8}\right)\cos M - \frac{e^2}{2}\cos 2M - \frac{3}{8}e^3\cos 3M - \ldots \tag{19}$$

or its inverse proportion:

$$\frac{a}{r} = 1 + e\left(1 - \frac{e^2}{8}\right)\cos M + e^2\cos 2M + \frac{9}{8}e^3\cos 3M + \ldots \tag{20}$$

Let us go next to a discussion connected with the position of the satellite in space. In Diagram 2, let point S designate the momentary position of the satellite; B is a pole of the earth; the sign $\delta b$ is the place where the satellite orbit crosses the equatorial plane; P is the perigee; the sign $\mathcal{V}$ is the point of vernal equinox (all points along the orbit projected onto the surface of the earth). Designating the angles and sides of the triangle $SS'\delta b$ with u, $\delta_s$, and $\Delta\lambda$ and i respectively, we are easily able to write the formulas resulting from

the elements of this (right angle)
triangle:

$$\sin \delta_s = \sin i \sin u \qquad (21)$$

$$\operatorname{tg} \Delta \lambda' = \cos i \operatorname{tg} u \qquad (22)$$

$$\operatorname{tg} \delta_s \operatorname{cosec} \Delta \lambda' = \operatorname{tg} u \qquad (23)$$

$$\cos \delta_s \cos \Delta \lambda' = \cos u \qquad (24)$$

It is not difficult to note that
$\Delta \lambda' = \lambda_s - \lambda_{sb} = a_s - a_{sb}$ ; and
between the geocentric declination of
the satellite $\delta_s$ and the geographic latitude of its projection on
the surface of the earth is the equality $\delta_s = \varphi_s$ .

From Diagram 2 we can also see that the satellite right ascension
$a_s$ is equal to the local sidereal time at the point beneath the satellite.
If we project the sputnik orbit onto the surface of the earth, we must
keep in mind the fact that the earth in the time $T_{sb} - T_s$ rotates on
its own axis from west to east at an angle of $\delta \lambda = \frac{2\pi}{1^d}(T_{sb} - T_s)$, during
which time the satellite has taken a path equal to arc u. Angle $\delta \lambda$
must be taken into account in solving problems dealing with the determina-
tion of the satellite geographic longitude.

Let us designate

$$\lambda_s - \lambda_{sb} + \frac{2\pi}{1^d}(T_{sb} - T_s) = \Delta \lambda_s + \delta \lambda = \Delta \lambda'' \qquad (25)$$

we obtain the relationships

$$\sin \varphi_s = \sin i \sin u, \qquad (26)$$

$$\Delta \lambda'' = \operatorname{arc} \operatorname{tg} (\cos i \operatorname{tg} u + \delta \lambda) \qquad (27)$$

$$\Delta \lambda'' = \operatorname{arc} \sin (\cot g\, i \operatorname{tg} \varphi_s + \delta \lambda) \qquad (28)$$

$$\Delta \lambda'' = \operatorname{arc} \cos (\cos u \sec \varphi_s + \delta \lambda), \qquad (29)$$

The value $T_{sb} - T_s$ can be computed with the help of Kepler's
equation (15).

Then we can write the equation indicating the relationship be-
tween the moving angular distance of the satellite from the terrestrial
equator, measured in the orbital plane, with time:

$$u = \frac{2\pi}{1^d}(T_{sb} - T_s) = \frac{2\pi}{1436^m}(T_{sb} - T_s)^m = \frac{2\pi N}{P}(T_{sb} - T_s), \qquad (30)$$

where 1436 is the number of sidereal minutes of the day, and N is the
number of satellite revolutions in the course of the day.

Let us now work out the
main parameters of the satellite's
elliptical orbit, which at the
same time define its shape and
dimensions as well as its spatial
orientation. They are: the
major semiaxis a, the minor
semiaxis b, the eccentricity

$$e = \frac{(a^2 - b^2)^{1/2}}{a},$$

the inclination of the orbital
plane to the plane of the
terrestrial equator i, the right
ascension of the ascending node
$\mathcal{V}_{\mathfrak{N}} = \alpha_{\mathfrak{N}}$ (argument of perigee,
the time of satellite passage through the
ascending node $T_{\mathfrak{N}}$ or the time of its passage through perigee $T_p$),
period $P = \dfrac{2\pi a^{3/2}}{fm_2^{1/2}}$ (draconic $P_{\mathfrak{N}}$ , anomalistic $P_p$).

And since the incliation i and the right ascension of point $\alpha_{\mathfrak{N}}$
define the orientation of the orbital plane in space (Diagram 3), the
argument of latitude of perigee $\omega$ indicates the orientation of the
orbital ellipse in the orbital plane, while the eccentric e and major
semiaxis $a$ characterize the dimensions and shape of the orbit. The
time of satellite passage through the ascending node or through perigee
$T_{\mathfrak{N}}$ or $T_p$ and period P link satellite motion with time.

E.  Perturbation of the Orbit

The satellite orbit described and presented by formulas 1 - 30
would refer to the theoretical case of a satellite revolving around an
earth that is a homogeneous, isolated sphere. However, the satellite
flight occurs within the gravitational field of an earth, which, in
reality, is an oblate spheroid with a nonhomogeneous interior; moreover,
as is known, the earth system itself is but a sputnik in the sphere of
influence of other bodies of the solar system, such as the moon, sun,
and planets. As a result, both the spatial orientation of the sputnik's
orbit, as well as its shape and dimensions, are subjected to secular
and period changes, i.e., perturbations. The greatest perturbations
in a sputnik's orbit result from the following causes:
1.  The oblateness of the earth causes significant secular changes
in the value of the right ascension of the ascending node (motion of
line of nodes), $\Delta a_{\mathfrak{N}}$ , and significant changes in the value of the
argument of perigee (motion of the line of apsides) $\Delta \omega$; then it influences
the change of the orientation of the orbit in space (for example, in
the case of sputnik 1958 $\delta$  the change of $\Delta \omega$ for each 24-hour period

was about 0.4°); it also causes very small periodic changes. It influences in a very insignificant way change in the shape and dimensions of the orbit. Perturbation is a function of the oblateness of the earth and of the inclination of the satellite orbit to the terrestrial equatorial orbit.

2. Air resistance effects significant changes in satellite velocity, lessening of sputnik distance from the center of the earth (height at perigee), lessening of P (and consequently of $\mathscr{A}$ ) and lessening of eccentricity e. It does not evoke changes in the orientation of the orbit in space, but rather secular changes in the shape and magnitude of the orbit. It is a function of the distance of the sputnik from the surface of the earth.

3. The remaining sources of orbital perturbation are the following:
  (a) the gravitational influence of the moon, sun, and planets;
  (b) the assymetry of the earth and the nonhomogeneity of its structure;
  (c) oncoming frictions;
  (d) the rotational motion of the satellite;
  (e) the mutual forces of the electromagnetic and electrostatic fields;
  (f) the rotation of the atmosphere;
  (g) the effects of the theory of relativity expressed in the motion of perigee.

The effects named in paragraph c are considerably smaller than the results of the force of the earth's oblateness and of atmospheric resistance, and are difficult to determine quantitatively, both from the point of view of their small value and the difficulty of computing them. We will not give them much attention, since the practical assumptions of the example developed in part two operate with accuracies many times smaller. We will take some time, however, on the effect of the first two agents in paragraphs a and b.

The problem of the motion of a satellite in the gravitational field of an oblate planet is basically not a new problem, since it was solved in relation to the theory of the motion of the moon of large planets and also in regard to the earth's moon. However, the orbits of artificial satellites are subjected to many special features which markedly differ from the orbits of the natural satellites known to date. The decisive role here is played by the great inclinations of the orbital planes of sputniks relative to the terrestrial equatorial plane, as well as their nearness to its surface. The present theories relative to satellites with a small angle of inclination and considerable distance from the mother planet can not be used in the matter of artificial earth satellites.

Atmospheric resistance is the main perturbation force for the motion of the sputnik around the center of mass for the first artificial satellites, considering their relatively small height at perigee. The moments of the gravitational forces of the earth become substantial only in relation to satellites having considerable height at perigee

and those having great height at perigee, and with sufficiently different moments of inertia. For artificial satellites having a height of 500 km and above, the aerodynamic forces are insignificant, and most important are perturbations of gravitational origin.

The theory of the motion of an artificial satellite under the action of gravitational forces leads to the following relationships of secular changes in the elements of the orbit (15):

$$\frac{dw}{dt} = \frac{A}{2p^2} \ (4 - 5\sin^2 i) + \frac{1}{10p^4} \left[ \left( \frac{24}{7} + \frac{27e^2}{7} - \frac{9e^2 \cos 2w}{7} \sin^2 i \right) \frac{93}{7} - \right.$$

$$- \frac{9 \cos 2w}{7} + \frac{27e^2}{2} - \frac{105e^2 \cos 2w}{14} (+\sin^4 i) \frac{21}{2} - \frac{81e^2}{8} -$$

$$\left. - \frac{3\cos 2w}{2} - \frac{27e^2 \cos 2w}{2} \right] , \tag{31}$$

$$\frac{da_{SL}}{dt} = - \frac{\cos i}{p^2} \left[ A + \frac{2B}{5p^2} \left( \frac{3}{7} + \frac{9e^2}{14} - \frac{9e^2 \cos 2w}{28} \right) - \frac{2B \sin^2 i}{5p^2} \left( \frac{3}{4} + \right. \right.$$

$$\left. \left. + \frac{9e^2}{8} - \frac{3e^2 \cos 2w}{4} \right) \right] , \tag{32}$$

$$\frac{da}{dt} = 0$$

$$\frac{de}{dt} = \frac{e \ \sin^2 i \sin 2w \ B \ (1 - e^2)}{20p^2} \left( \frac{18}{7} - 3 \sin^2 i \right)$$

$$\frac{di}{dt} = - \frac{3Be^2 \ \sin 2i \ \sin 2w \ (6 - 7 \sin^2 i)}{280p^4}$$

where: $w$  is the argument of perigee;

$a_{SL}$  is the right ascension of the ascending node;

$a$  is the major semiaxis;

$e$  is the eccentricity of the satellite orbit;

$i$  is the inclination of the orbital to the equatorial plane;

$p$  is the parameter of the orbit = $a(1 - e^2)$;

while the descending factors A and B of the potential of the terrestrial spheroid expanded in series equa.: A = 0.001 641, B = 0.000 010 6.

Formulas (31) and (32), taking into account expression of the first order, may be put into a simplified form:

$$\frac{\Delta w}{t} = \frac{1}{2} \ A \left( \frac{a'}{a} \right)^2 \ \frac{5 \cos^2 i - 1}{(1 - e^2)^2} \ n, \tag{31'}$$

$$\frac{\Delta a_{SL}}{\Delta t} = - A \left( \frac{a'}{a} \right)^2 \ \frac{\cos i}{(1 - e^2)^2} \ n, \tag{32'}$$

where $a$ is the major semiaxis of the earth, $a'$ is the major semiaxis of the satellite orbit, $n = \frac{2\pi}{p}$ the mean 24-hour period motion of the satellite, while factor A is expressed by the formula $A = \varepsilon - \frac{\omega^2 (a')^3}{fm_2}$ , wherein $\varepsilon$ is the oblateness of the terrestrial ellipsoid, $\omega$ is here the angular velocity of the rotational motion of the earth.

Finally we are able to form still more simplified expressions:

$$\frac{\Delta \omega}{1\ \text{revolution}} = \frac{\pi \varepsilon R^2}{a^2 (1 - e^2)^2} \ (5 \cos^2 i - 1); \qquad (31")$$

$$\frac{\Delta \alpha_{\mathfrak{R}}}{1\ \text{revolution}} = - \frac{2 \pi \varepsilon R^2}{a^2 (1 - e^2)^2} \ \cos i; \qquad (32")$$

after substituting given numerical data we obtain:

$$\frac{\Delta \omega}{1\ \text{revolution}} = 0.294 N\ \frac{5 \cos^2 i - 1}{a^2 (1 - e^2)^2} \ ; \qquad (33)$$

$$\frac{\Delta \alpha_{\mathfrak{R}}}{1\ \text{revolution}} = - 0.588 N\ \frac{\cos i}{a^2 (1 - e^2)^2} \ ; \qquad (34)$$

N is the number of revolutions per 24-hour period.

From the three formulas 31 - 34 we can easily see that when $i = 0°$ or $180°$, the secular motion of the ascending node reaches maximum, while when $i = 90°$, $\Delta \alpha_{\mathfrak{R}} = 0$. The secular motion of the perigee is maximal for satellites having an inclination of $i \cong 63°\ 7^2$.

Further, we can present the formulas expressing the perturbation effects of the aerodynamic force (atmospheric resistance) on the orbit elements (16):

$$\frac{da}{dt} = - \frac{2 \alpha \rho n a^2}{(1 - e^2)^{3/2}} \ (1 + 2e \cos v + e^2)^{3/2} \ ; \qquad (35)$$

$$\frac{de}{dt} = - \frac{2 \alpha \rho n a^2}{(1 - e^2)^{1/2}} \ (1 + 2e \cos v + e^2)^{1/2} (e + \cos v); \qquad (36)$$

$$\frac{da_{\mathfrak{R}}}{dt} = 0;$$

where $\rho$ is the density of the atmosphere, $\alpha = 1/2 C \frac{S}{m}$ , C is the coefficient of aerodynamic resistance, S is the area of the transverse section of the satellite, m is the satellite mass.

The influence of the atmospheric resistance on the period of satellite revolution P, which value is connected with the major semiaxis of the orbit a by the relationship (12), may be expressed by the approximation formula (37), based on a knowledge of the secular change

of the major semiaxis a:

$$\Delta P = \frac{2}{3} \; \Delta a \; \frac{P}{a} \; . \tag{37}$$

Between $\Delta e$ and $\Delta a$ exists the relationship

$$\frac{\Delta a}{a} = \Delta e(\cos M - e \cos 2M...) \tag{38}$$

## PART II

## Example of Computing the Geographic Position and the Height Above the Earth of Artificial Earth Satellite 1958

### F. Problem Conditions

The Zegrze Observatory (Artificial Earth-Satellite Observation Station, Nr. 157) conducted a series of radio-monitoring observations of satellite 1958 2 (sputnik III) in the second half of May 1958. In the desire to obtain documental material to confirm the theory of radio-wave propagation in the ionosphere (5), it was necessary to compute satellite position expressed in geographic coordinates and its height above the surface of the earth and distance along the orthodrom from points beneath the satellite to the Zegrze station for the observation moments given below (start of signal, maximum, and end). This example presents computations dealing with selected observations shown in Table 1.

TABLE 1

| Nr. obs. | Date | Time UT | |
|---|---|---|---|
| 53 | 22 May 58 | $8^h$ | $36^m$ |
| | | | 38 |
| | | | 39 |
| 54 | 22 May 58 | 10 | 21 |
| | | | 25 |
| | | | 27 |
| 84 | 24 May 58 | 15 | 30 |
| | | | 32 |
| | | | 36 |
| 99 | 25 May 58 | 16 | 15 |
| | | | 16 |
| | | | 20 |
| 126 | 27 May 58 | 14 | 02 |
| | | | 04 |
| | | | 10 |

### G. Initial Materials

To solve the problem defined in paragraph 6 with the aid of formulas 14-30, the values of the orbital parameters named at the end of section 4 referring to the observation epoch of the Zegrze Station radio monitoring, as well as the values of the secular changes of these parameters, must be at hand. As a result of surveying all possible documents, it was possible to gather the following data:

$$i = 65^\circ \quad \text{(Circ. Bureau Centr. de telegr. astr. Kopenhaga)}$$

$$P = 105.^{m}52 \quad t_{o} = VI. 20^{c} \quad \text{(IGY World Data Center A Rocket and Satellites Nr. 6)}$$

$$\Delta P/1^{d} = -0.^{m}011 \qquad \text{"}$$

$$\omega = 49.6 \qquad \text{"}$$

$$\Delta \omega/1^{d} = 0.317 \qquad \text{"}$$

$$r_{p} = 658.3 \text{ km} \qquad \text{"}$$

$$H_{p} = 187.3 \text{ km} \qquad \text{"}$$

$$H_{A} = 1880 \text{ km} \qquad \text{"}$$

$$e = 0.112 \qquad \text{"}$$

where i is the inclination of the orbit, P is the anomalistic period, $\omega$ is the argument of perigee, $r_p$ is the distance from the center of the earth to the perigee, $H_p$ is the height above the surface of the earth to perigee, $H_A$ is the height above the earth to apogee, e is the eccentricity of the orbit, $t_o$ is the epoch for which the above values are given.

As seen from the above compilation, the material gathered was not complete, values for the changes in the time of elements e, i, r, values for the right ascension of ascending node as well as any element linking the satellite position to time, are missing. The problems of computing the missing data on the basis of observational material arose. Unfortunately, no position observations were made in Poland during the period in which the radio observations were made (second half of May through beginning of June). Sputnik III was not visible at night over Polish territory from the moment of launching (15 May 1958) until the middle of June; then, however, a period of bad weather set in. Computation of the missing orbit elements on the basis of observations from the end of June through July could lead to major errors due to extrapolation. In view of this foreign data, published in one of the issues of "IGY World Data Center A, Rockets and Satellites" (8) were relied upon. From the lists of observations given there, carried out in May 1958 by observation stations of different continents, those cited in Table 2 were chosen.

TABLE 2

| Lp. | Station | Time UT | $\alpha$ | $\delta$ | $a_n$ | h |
|---|---|---|---|---|---|---|
| 1 | Johannesburg* | 23 May $03^h$ $36^m$ $20^s$ | $02^h55^m$ | $-42°00'$ | | |
| | " | 23 May 03 36 23 | 02 54 | $-41$ 30 | | |
| 2 | Pretoria | 23 May 03 36 52 | 02 49 | $-36$ 12 | | |
| 3 | Capetown | 27 May 18 05 25 | | | $0°24'$ | $53°30'$ |
| 4 | Sydney | 27 May 18 44 32.5 | 23 44 | $-30$ 30 | | |
| 5 | Pretoria | 29 May 02 27 40 | 02 18 | $-69$ 00 | | |
| 6 | Pretoria | 29 May 02 30 27 | 01 28 | $-30$ 18 | | |
| 7 | Sacramento* | 14 Jun 11 37 24.9 | 22 23 47 | $-17$ 02 | | |
| | " | 14 Jun 11 37 36.0 | 23 02 53 | $-08$ 01 | | |
| 8 | Walnut Creek | 14 Jun 11 37 28.6 | 22 14 18 | $-08$ 45 | | |
| | " | 14 Jun 11 37 44.3 | 23 02 18 | $-07$ 36 | | |
| 9 | Sacramento* | 14 Jun 11 32 18.7 | 01 37 59 | $+52$ 32 | | |
| 10 | Walnut Creek | 14 Jun 11 39 05.2 | 03 54 30 | $+39$ 30 | | |
| 11 | Oakland | 14 Jun 13 29 59 | 02 65 | $+25$ | | |
| 12 | Bryn Athyn | 17 Jun 08 16 29 | 19 10 | $+68$ | | |
| 13 | Cambridge | 17 Jun 08 16 51 | | | 273 30 | 35 30 |
| 14 | Cambridge | 17 Jun 08 17 52 | | | 1 18 | 29 30 |

*For computations the mean from two observations was taken.

The geographic coordinates of the stations are given in Table 3.

TABLE 3

| Lp. | Station | $\lambda_E$ | $\varphi_N$ |
|---|---|---|---|
| 1 | Bryn Athyn (USA) | $284°56'$ | $+40°08'$ |
| 2 | Cambridge (USA) | 288 52 | $+42$ 23 |
| 3 | Capetown (Unia Pld.-Afr.) | 18 02 | $-33$ 56 |
| 4 | Johannesburg (Unia Pld.-Afr.) | 28 04 | $-26$ 11 |
| 5 | Oakland (USA) | 237 48 | $+37$ 47 |
| 6 | Pretoria (Unia Pld.-Afr.) | 28 12 42".58 | $-25$ 43 42".56 |
| 7 | Sacramento (USA) | 238 14 51.0 | $+38$ 32 55.8 |
| 8 | Sydney (Australia) | 151 06 | $-33$ 55 |
| 9 | Walnut (USA) | 237 56 14 | $+37$ 57 52 |

The materials above include then the station geographic coordinates, date, hour, minute, second, and its fractional part at the moment of observation, as well as the topocentric equatorial coordinates $\alpha$ and $\delta$ and a and h of the position of the observed sputnik.

## H. Solution

Solution of the problem under the conditions described led to the following successive computational and analytical steps:

1. Computation by way of successive approximations of the geographic coordinates $\varphi$, $\lambda$ of subsatellite points for the individual astronomical observations, listed in Table 2. In accomplishing this stage, formulas 18-20, 26, and derivatives were used. To compute the angular distance of the subsatellite point from the observational station, formula (41), developed by W. Opalski (11), helped. These examples illustrate this part of the computations.

Observation nr. 5, Pretoria V. $29^d 02^h 27^m 40^s$

$$\alpha = 02^h 18^m \qquad \varphi = -25°43'44''$$
$$\delta = -68°00' \qquad \lambda = 28°12'43''$$

The first function was the recomputation of observed, topocentric, equatorial coordinates on the horizontal with the help of formulas:

$$\cos z = \sin\delta \sin\varphi + \cos\delta \cos\varphi \cos t$$

$$-\sin q_n = \frac{\cos\delta \sin t}{\sin z}$$

In the given case we obtain

$$q_n = 156°36'30'' \qquad z = 63°41'30''$$

Next we compute in the first approximation (Diagram 5) the coordinates of the subsatellite point with the formulas:

$$\lambda'_s = \lambda_{pr} + \Delta\lambda \ , \quad \varphi'_s = \varphi'_{pr} + \Delta\varphi \ ,$$

$$\Delta\varphi = 1 \cdot \cos q_n \cdot \frac{3438'}{R} \ ,$$

$$\Delta\lambda = 1 \sin q_n \cdot \frac{3438'}{R \cos\varphi}$$

where R = 6371 is the mean radius of the earth,

$$1 = H \cdot ctg \ h,$$

while

H is computed with the formulas
$$H = r - R,$$

where

$$r = a \left[ 1 - e \cos M - \frac{e^2}{2} (\cos 2M - 1) \dots \right] \ ; \qquad (39)$$

the major semiaxis directly for the initial data:

$$a = \frac{R + H_A}{1 + e} = \frac{R + H_p}{1 - e} = 7403 \text{ km}^3$$

$$M = u - \omega - 2e \sin M, \tag{40}$$

$$\sin u = \sin \varphi \operatorname{cosec} i.$$

Numerically, this is shown as follows:

$$\omega_v \ 29 = 49°6 + 22^d \ 0.317 = 56°6.$$

Since the coordinates of the subsatellite point are not yet known, we take in the first approximation

$$\varphi_s \cong \varphi \text{Pret.}$$

as if the observation had been made in the zenith. Then

$$\sin u = \sin(-25°7) \operatorname{cosec} 65° = -0.434 \cdot 1.103 = -0.479$$
$$u = -28°6.$$

Basically, M also had to be computed by the method of successive approximations from equation (40). However, considering the approximate character of the entire computation, in this part, to find sin 2M, it is possible to take the value M resulting from the first two member of the formula alone, and correcting for the probable influence of the third expression. Thus it will be:

$$M = -28°6 - 56°6 + 0.224 \cdot 0.956 \cdot 3438' = -73°0,$$
$$r = 7403 \ [1 - 0.112 \cdot 0.293 - 0.0063 \ (-0.8284 - 1)] = 7242 \text{ km},$$
$$H = 7242 - 6371 = 871 \text{ km},$$
$$l = 871 \cdot 2.0216 = 1761 \text{ km},$$
$$\Delta\varphi_s = -1761 \cdot 0.9178 \cdot 0.5396 = -14°5$$
$$\Delta\lambda_s = 1761 \cdot 0.3970 \ \frac{0.5396}{0.9008} = +7°0,$$

whence

$$\varphi_s \cong 39° .$$

We get the second approximation by solving the same problem for a sphere and instead of using longitude l, mean angle $\zeta$ which is computed with the aid of the following formula is used:

$$\operatorname{tg} \zeta = \frac{\sqrt{1 + (1 + \operatorname{ctg}^2 h) \ \frac{2H}{R} - 1}}{\operatorname{tg} h + \operatorname{ctg} h} \tag{41}$$

At the same time, using the same method as previously, height H must be computed again:

$$\sin u = -0.62932 \cdot 1.1034 = -0.69439,$$

$$u = -43^\circ 59',$$

$$M = -87^\circ.46,$$

$$r = 7464 \text{ km},$$

$$H = 7464 - 6371 = 1093 \text{ km}.$$

Substituting these values in formula (41), we obtain

$$tg \zeta = \frac{\sqrt{1 + (1 + 4.08687) \cdot 0.34312} - 1}{2.11600} = 0.26110,$$

$$\zeta = 14^\circ 38',$$

$\varphi_s$ and $\lambda_s$ are finally computed with the formulas

$$\sin \varphi_s = \cos \zeta \sin \varphi_{Pret.} + \sin \zeta \cos \varphi_{Pret.} \cos \alpha_n$$

$$\sin \Delta \lambda_s = \frac{\sin \alpha_n \sin \zeta}{\cos \varphi_s}$$

After substituting numbers, we have

$$\sin \varphi_s = -0.96756 \cdot 0.43411 - 0.25263 \cdot 0.90085 \cdot 0.91781 = -0.62813,$$

$$\varphi_s = 38^\circ 55' S,$$

$$\sin \Delta \lambda_s = \frac{0.39701 \cdot 0.25263}{0.77806} = +0.12891,$$

$$\sin \Delta \lambda_s = +7^\circ 24'$$

$$\lambda_s = 35^\circ 37' E.$$

In certain cases, when the satellite observation is made near perigee, a still more simplified method of computing the subsatellite point is applied. Considering the relatively small height (about 200 km), and the consequent small distances l, solution can be limited for the plane. A reduction of observation nr. 12 is given for an example:

Bryn Athyn VI. $17^d 08^h 16^m 29^s$; $\alpha = 19^h 10'$ $\delta = +68$

$$\varphi_{BA} = +40^\circ 08', \quad \lambda_{BA} = 284^\circ 56'.$$

After computing the equatorial coordinates on the horizontal, we obtain

$$\alpha_n = 341^\circ 11', \quad h = 58^\circ 37'.$$

Proceeding in the previous way, we compute

$$\omega_{v\ 17}^{d} = 49^{\circ}6 + 3^{d}.0.317 = 50^{\circ}55 = 50^{\circ}33',$$

$$\sin u = 0.6446 \cdot 1.1034 = 0.7119,$$

$$u = 45^{\circ}23',$$

$$M = + 45^{\circ}23' - 50^{\circ}33' + 0.224 \cdot 0.070 \cdot 3438' = -4^{\circ}15',$$

$$r = 7403\ (1 - 0.112 \cdot 0.9973) = 6577\ \text{km}.$$

In the last substitution, owing to the small value M, the third term is discarded.

$$H = 6577 - 6371 = 206\ \text{km},$$

$$1 = 206 \cdot 0.6100 = 126\ \text{km},$$

$$\Delta \varphi_{s} = \frac{126 \cdot 0.94656 \cdot 3438'}{6371} = +1^{\circ}04',$$

$$\Delta \lambda_{s} = \frac{-126 \cdot 0.32251 \cdot 3438}{6371 \cdot 0.76455} = -0^{\circ}29'.$$

Having the above, we immediately compute $\varphi_{s}$ and $\lambda_{s}$ with sufficient accuracy:

$$\varphi_{s} = 41^{\circ}12',\qquad \lambda_{s} = 284^{\circ}27'.$$

In the method described above everything has been reduced, the observations placed in Table 2: the results given in table 4 are taken.

| Lp. | Station | Date | $\varphi$ | $\lambda$ (E) |
|---|---|---|---|---|
| 1 | Joh. | V. 23$^{d}$ | -37°21' | 43 44' |
| 2 | Pret. | V. 23 | -33 57 | 44 53 |
| 3 | Cap. | V. 27 | -20 59 | 18 03 |
| 4 | Syd. | V. 27 | -34 13 | 157 18 |
| 5 | Pret. | V. 29 | -38 55 | 35 37 |
| 6 | Pret. | V. 29 | -36 18 | 53 04 |
| 7 | Sac. | VI. 14 | +36 34 | 239 54 |
| 8 | W. Cr. | VI. 14 | +35 54 | 239 35 |
| 9 | Secr. | VI. 14 | +39 04 | 242 14 |
| 10 | W. Cr. | VI. 14 | +45 54 | 251 04 |
| 11 | Oak. | VI. 14 | +38 12 | 238 32 |
| 12 | Br. At. | VI. 17 | +41 12 | 284 27 |
| 13 | Cam. | VI. 17 | +42 13 | 285 19 |
| 14 | Cam. | VI. 17 | +45 41 | 288 58 |

2. Computation of the missing orbit elements on the basis of the results of the first stage and by taking into account initial data for separate moments of astronomical observations, i.e.: computations for a certain epoch of the moment of satellite passage through ascending node $T_{\Omega}$ ; computation, for the same epoch, of the geographic longitude of ascending node $\lambda_{0}$; and further computation for a certain epoch

of the value of right ascension of the ascending node $\Omega$. In addition, in this state of work the changes following are computed: $\Delta \lambda_o/1$ revolution, $\partial \lambda_o/(1 \text{ revolution})^2$, and $\Delta \Omega/1^d$.

To realize this problem, a passage from the satellite position at the moment of observation to its position (longitude) at the moment of passage through the equatorial plane, and from the moment of observation $T_i$ to the moment of passage through the ascending node $T_{\Omega i}$, is necessary.

We use the formulas:

$$T_i - T_{\Omega \ i} = \frac{u}{360^o} \ P^m, \tag{42}$$

where P is the period, while $\sin u = \sin \varphi_s \ \mathrm{cosec} \ i$

$$\lambda_s - \lambda_{oi} = \Delta \lambda' - \Delta \lambda'' \tag{43}$$

The first expression of the right side is computed by the formula

$$\sin \Delta \lambda' = \mathrm{tg} \ \varphi \ \mathrm{ctg} \ i. \tag{44}$$

The second expression results from the revolving motion of the earth and is expressed by the relationship:

$$\Delta \lambda'' = \frac{u}{360^o} \ P^o \tag{45}$$

where

$$P^o = \frac{P^m}{4} \ .$$

The course of computations on the example of observation nr. 12:

$$\sin \Delta \lambda' = + \ 0.87543 \cdot 0.46631 = + \ 0.40822,$$
$$\Delta \lambda' = + \ 24^o06',$$
$$\sin u = + \ 0.65869 \cdot 1.10338 = + \ 0.72679,$$
$$u = + \ 46.7,$$
$$\frac{u}{360^o} = + \ 0.12972,$$
$$P^o = \frac{1}{4} \ [105.52 - 0.011 \ (17^d - 20^d)] = 26^o39.$$

Substituting in (45), we have

$$\Delta \lambda'' = + \ 0.12972 \cdot 26^o39 = + \ 3^o25',$$

whence

$$\lambda_{oi} = 284^o 17' + 24^o06' + 3^o25' = 362^o46'.$$

Applying the above described progression to all observations, the results listed in Table 5 are obtained.

TABLE 5

| Nr. of Pass. Lp. | $\lambda_{oi}$ | $\lambda_{oi}$ mean | Nr. of Pass. Lp. | $\lambda_{oi}$ | $\lambda_{oi}$ mean |
|---|---|---|---|---|---|
| 1 | 87°57' ⎫ | 87.38 | 8 | 222°49' ⎫ | 222.92 |
| 2 | 86 48 ⎭ | | 9 | 223 16 ⎬ | |
| 3 | 202 40 ⎫ | 201.40 | 10 | 226 09 ⎬ * | |
| 4 | 200 09 ⎭ | | 11 | 220 10 ⎭ * | |
| 5 | 79 56 ⎫ | 79.94 | 12 | 263 46 ⎫ | |
| 6 | 96 32 ⎬ * | -- | 13 | 263 48 ⎬ | 263.95 |
| 7 | 222 41 ⎭ | | 14 | 264 15 ⎭ | |

The results contained in brackets refer to the same moments $T_{\Omega i}$ (close observations in time during the same revolution), need not differ from each other. Discrepancies arise from observational errors. Results bearing gross errors are discarded, while means are formed from the remaining.

If passage from $\lambda_{oi}$ averaged to $\lambda_o$ for a certain selected epoch, the former would have to be brought to some common system of reference. To do this, it would be necessary to compute the number of complete revolutions made by the satellite between separate epochs $T^{\Omega}$. The values $T_{\Omega i}$ are computed according to formula (42) with accuracy up to $1^m$; further computations are shown in Table 6.

TABLE 6

| Nr. of Passage | $T_i$ | $T_i - T_{\Omega i} =$ $= \dfrac{u}{360} P^m$ | $T_{\Omega i}$ | Time interval between successive moments $T_{\Omega i}$ |
|---|---|---|---|---|
| 1 | V. 23$^d$ 3$^h$36$^m$ | 1$^h$33$^m$ | 2$^h$03$^m$ ⎫ 2$^h$03$^m$ | |
| 2 | 3 37 | 1 34 | 2 03 ⎭ | 111$^h$04$^m$ = 6664$^m$ |
| 3 | V. 27 18 05 | 0 59 | 17 06 ⎫ 17 07 | |
| 4 | 18 44 | 1 37 | 17 08 ⎭ | 31$^h$48$^m$ = 1908$^m$ |
| 5 | V. 29 2 28 | 1 33 | 0 55    0 55 | |
| 6 | 2 30 | -- | | |
| 7 | VI. 14 11 37 | 0 12 | 11 25 ⎫ | 394$^h$30$^m$ = 23670$^m$ |
| 8 | 11 38 | 0 13 | 11 25 ⎬ 11 25 | |
| 9 | 11 38 | 0 13 | 11 25 ⎭ | |
| 10 | 11 39 | -- | | |
| 11 | 13 30 | -- | | 68$^h$38$^m$ = 4118$^m$ |
| 12 | VI. 17 8 16 | 0 14 | 8 02 ⎫ | |
| 13 | 8 17 | 0 14 | 8 03 ⎬ 8 03 | |
| 14 | 8 18 | 0 15 | 8 03 ⎭ | |

The number of complete revolutions $n = \dfrac{\text{time interval}}{p^m}$ in specific

intervals is 63, 18, 224, and 39 respectively.

Having the value n, we can compute the intervals expressed in a gradual measure between separate values of $\lambda_{oi}$. They are, respectively:

$$1685\overset{o}{.}98$$
$$481.46$$
$$5977.02$$
$$1038.97.$$

Dividing them by successive n, we obtain $\Delta\lambda_o$:

$$-26\overset{o}{.}76$$
$$-26.75$$
$$-26.68$$
$$-26.64 \ .$$

As we see, values $\Delta\lambda_o$ show distinct changes in time which agrees with theory. To bring $\lambda_{oi}$ and $\Delta\lambda_o$ to a single epoch, smoothing is performed, involving the following equation of error:

$$\lambda_o + n\cdot\Delta\lambda_o + n^2\cdot\ \partial\lambda_o - \ \lambda_{oi} = v.$$

As epoch, the moment of the first sputnik passage through the ascending node on VI. $1^e \ 0^h$ is chosen, and from it, the entire number of revolutions n for separate moments $T_{\lambda_i}$ is computed. This choice os epoch was founded by expansion in time of both the initial astronomical observations as well as the radio observations. The values $\lambda_{oi}$ (averaged) are compiled in Table 5, corrected for the entire number of complete angles so that the zero of an obtained numerical axis could be found near the selected epoch. A system of equations of errors is obtained and then a system of normal equations, which are solved by the method of krakovian roots (Table 7).

TABLE 7

| No. of Pass. | a | b | c | l | s |
|---|---|---|---|---|---|
| 5 | 1 | ÷222 | ÷49284 | ÷5 856.05 | ÷55 363.05 |
| 4 | 1 | +183 | ÷33489 | ÷4 817.08 | +38 490.08 |
| 3 | 1 | - 41 | + 1681 | -1 159.94 | ÷ 481.06 |
| 2 | 1 | - 59 | + 3481 | -1 641.40 | + 1 781.60 |
| 1 | 1 | -122 | +14884 | -3 327.38 | +11 435.62 |

| | a | b | c | l | s |
|---|---|---|---|---|---|
| a | ÷5 | ÷ 183 | ÷ 102 819 | ÷ 4 544.41 | ÷ 107 551.41 |
| b | | ÷102 819 | ÷ 14 979 387 | ÷ 2 731 909.24 | ÷ 17 814 298.24 |
| c | | ÷102 819 | ÷3 786 902 355 | ÷392 740 463.86 | ÷4 194 725 024.86 |

| | | | | | |
|---|---|---|---|---|---|
| ÷2,235 907 | ÷81,846 | ÷ 45 985.30 | ÷ 2 032.47 | ÷ 48 101.92 | |
| | ÷310,032 | ÷ 36 175.86 | ÷ 8 275.14 | ÷ 44 761.02 | |
| | | ÷ 19 067.31 | - 4.38 | ÷ 19 062.68 | |

$$÷ 64.276 \qquad -26.7180 \qquad ÷0.00023$$

$$\overset{a}{\lambda}_0 = 64°276,$$

$$\Delta \lambda_0 = -26.7180,$$

$$\partial \lambda_0 = ÷0.000\ 230.$$

The value of change $\Delta \lambda_0$ computed here contains two members: one is controlled by the earth's revolution, the second by the change in right ascension of the ascending node. The existence of the term $\partial \lambda_0$ is caused by a shortening of period P as a result of air resistance. It must be added that the function according to which the geographic longitude of the ascending node changes is not only very complicated, but is actually unknown. Therefore, the equation of error given here is only a certain approximation in which time sector works sufficiently well.

The following work computes the same epoch of value $T_{\Omega}$. We obtain this by multiplying P by the number of revolutions n, and by adding or subtracting from $T_{\Omega}$ i (Table 8).

## TABLE 8

| No. of Pass. | $P_i$ | $nP_i = \Delta T_i$ | $\Delta T_i^d$ | $T_{\Omega\,i}^d$ | $T_{\Omega}$ |
|---|---|---|---|---|---|
| 1 | $105^m.790$ | $+215^h06^m.38$ | $+8^d.962\ 75$ | $V.23^d.085\ 41$ | $VI.\ 1^d.048\ 16$ |
| 2 | 772 | $+104\ 00.55$ | $+4.333\ 70$ | $V.27.713\ 33$ | $1.047\ 03$ |
| 4 | 756 | $+\ 72\ 16.00$ | $+3.011\ 12$ | $V.29.038\ 33$ | $1.049\ 45$ |
| 3 | 661 | $-322\ 15.96$ | $-13.427\ 77$ | $VI.14.475\ 83$ | $1.048\ 06$ |
| 5 | 645 | $-390\ 53.19$ | $-16.286\ 91$ | $VI.17.335\ 41$ | $1.048\ 50$ |

$$\text{sr. } 1^d.048\ 24$$

After computation, we have

$$T_{\Omega} = VI \cdot 1^d 01^h 09^m 28^s.$$

It must be noted that in computing $T_{\Omega\,i}$ (see Table 6), accuracy is limited to minutes, while observational data yields seconds. However, the actual - considerably less - accuracy of observations and the relatively approximate formulas yield greater discrepancies than those resulting from rounding off. The accuracy demanded in the problem is nonetheless preserved.

The next and last step to obtain all the required orbit elements was the computation of the value of the right ascension of the ascending node and its daily changes. Our approach here is similar to that in previous computations of $l_o$ and $T_{\Omega}$. This means we compute $\alpha_{\Omega i}$ of successively individual initial observations, and then, having them expanded in time, we bring them to a single epoch and compute the change. To realize the above, we use the relationships adduced in part one:

$$\lambda_s - \lambda_{\Omega} = \alpha_s - \alpha_{\Omega}$$

and

$$\alpha_s = \Theta_s^{\#}.$$

The last equality is important for the point at which the satellite is located at a given moment, but for each point on the celestial sphere and its median projection onto the terrestrial sphere. In view of this, the right ascension of the ascending node at moment T will be equal to the sidereal time of place, above which, on the sphere at moment T, the orbital plane intersects the equatorial plane. This would be equal to

$$\Theta_{\Omega}^{\#} = \Theta_{Gr}^{\#} = \lambda_s - \Delta\lambda' = \alpha_{\Omega}$$

We find $\Theta_{Gr}^{\#}$ easily, having the given moment of observation in universal time. The sum of the two following terms is the geographic longitude of the ascending node at the moment of observation and was already computed in an earlier phase of calculations.

Here are the results of computations made according to the formulas above for observation nr. 12:

$$TU = 8^h16^m29^s,$$

$$\Theta^*_{Gr} = 1^h56^m48^s.7 = 29°12' \, 10'',$$

$$\lambda_s = 41°12',$$

$$\Delta\lambda' = 24°06',$$

$${}^a\Omega_i = 289°33'.$$

Computations of the remaining observations gave the results compiled in Table 9.

### TABLE 9

| No. of Pass. | ${}^a S_{c_i}$ | | No. of Pass. | ${}^a\Omega_i$ | |
|---|---|---|---|---|---|
| 1 | 358°55' | } | 8 | 296°31' | } |
| 2 | 357 36 | } | 9 | 296 50 | } |
| 3 | 344 19 | } | 10 | 299 20 | } |
| 4 | 341 44 | } | 11 | 321 50 | } |
| 5 | 339 45 | } | 12 | 289 33 | } |
| 6 | 357 40 | } | 13 | 289 35 | } |
| 7 | 296 18 | ) | 14 | 289 59 | } |

The results contained in brackets refer to near epochs and should differ slightly from each other. Some of them (6, 10, and 11) depart significantly from the remaining, indicating the presence of gross errors in observation, and these were eliminated in further computations. The remaining ones in individual groups are averaged, and epochs assigned to them are equal to the mean of the moments of observations. In this way we obtain Table 10.

### TABLE 10

| No. of. Pass. | $T_i$ | ${}^a\Omega_i$ |
|---|---|---|
| 1 | V.23$^d$.1504 | 358°.2584 |
| 2 | V.27.7671 | 342.8083 |
| 3 | V.29.1025 | 339.7500 |
| 4 | VI.14.4846 | 296.5500 |
| 5 | VI.17.3454 | 289.7167 |

Similarly, as in computing value $\lambda_o$, smoothing by the method of least squares is applied. The following form of the equation of error is obtained:

$$\alpha_{\Omega_o} + \Delta t \cdot \Delta \alpha_{\Omega} - \alpha_{\Omega_i} = v.$$

After resolving the system of normal equations, the following required unknowns are obtained:

$$\alpha_{\Omega_o} = 332°85; \qquad \Delta \alpha_{\Omega} = -2°63/1^d$$

for the epoch 1958 VI $1^d.0$.

It is known, however that the value $\Delta \alpha_{\Omega}$ can be computed independently of observation, relying on the known value of the oblateness of the earth, according to formula (32").

The value obtained in this way

$$\Delta \alpha_{\Omega} = -2°51/1^d.$$

Analyzing this discrepancy in results, it must be realized that the mathematical formulas used here in computations do not either in the first or in the second case describe accurately the physical conditions under which the investigated phenomena occur. On the one hand, the change in right ascension of the ascending node does not run linearly, but according to a very complicated function, while, on the other hand, the formula given above considers only one agent from among the many named in the first part that influence satellite motion. After thorough analysis, it was decided to take for further computations the rounded value equal to:

$$\Delta \alpha_{\Omega} = -2°6.$$

3. Already knowing all the required space-time orbit elements, it is possible to proceed to the second part of the problem, i.e., to computation of the geographic coordinates of the sputnik at the moments given in Table 1. The first phase of this computation, and third state of the whole, will be the extrapolation of the value of the orbit elements for required moments. This calculation is cited in Table 11, whose successive columns we will consider:
Col. 1 and 2 give the date and place of observation;
Col. 3 and 4 contain computation of $\omega_i$, according to formula:

$$\omega_i = 49°6 - 0°317 (t_i - t_o)^d \quad t_o = VI. \ 20^d5;$$

as indicator i, we shall now designate the processing of a given value of the i-th radio observation;
Col. 5 contains computation of period Pi according to formula:

$$Pi = 105^m.52 - 0.011 (t_i - t_o)^d;$$

Col. 6 contains computation of mean notion N

$$N = \frac{360°}{P^m} \quad ;$$

Col. 7 - 13 contain solution by the method of successive approximation of the equation

$$- N \cdot x = \dot{\omega} + 2e \cdot \sin Nx + \frac{5}{4} e^2 \sin 2Nx, \qquad (46)$$

where

$$N \cdot x = N(T_{\Omega} - Tp);$$

successive series approximations are in a verticle direction ; as first approximation $Nx = \omega$ is adduced. For observations 53 and 54 made on the same day a single common computation is made;

Col. 14 contains computation of the nodal period $P_{\Omega}$

$$P_{\Omega} = P + 0.0065;$$

Col. 15 and 16 contain the result of multiplication $P_{\Omega}$ x the number of complete revolutions (computed in col. 23);

Col. 17 contains the value $T_{\Omega\,i}$ obtained by subtracting (16) from $T_{\Omega} = VI.\ 1^d 01^h 09^m 5;$

Col. 18 gives, for the purpose of simplifying further computations, $T_i$ expressed in days;

Col. 19, 20. and 21 contain computation of the right ascension of the ascending node for the moment of observation;

Col. 22-27 contain computations of $\lambda_{oi}$ for revolutions corresponding to moments of observation;

Col. 28 gives the sidereal time of satellite passage through the ascending node:

$$\mathcal{E}^*_{\Omega\,i} = \Omega_i - \bar{\lambda}_{oi}$$

However, since $\alpha_{\Omega\,i}$ was computed for the moment of observation, it is necessary to correct for the difference $T_i - T_{\Omega\,i}$;

Col. 29 contains the difference $T_i - T_{\Omega\,i}$ expressed in days;

Col. 20 in the second section contains corresponding corrections of the right ascension;

Col. 30 contains $T_{\Omega\,i}$ computed for a second time by calculation with sidereal time (taking into account the noted corrections);

Col. 31 contains averages from (17) and (30);

Col. 32 gives the time of passage through perigee $T_p = (31) - (13)$.

TABLE 11

| Nr. of obs. | Date | $T_i$ | $(T_i - T_o)^d$ $T_o = VI.20^d5$ | $\dot{\omega}_i$ | $P_i$ | $N = \dfrac{360}{P_i}$ |
|---|---|---|---|---|---|---|
| 1 | | 2 | 3 | 4 | 5 | 6 |
| 53 | V.22$^d$ | 8$^h$36$^m$ 38 39 | -29.2 | 58.86 | 105.84 | 3.4014 |
| 54 | V.22 | 10 21 25 27 | -29.1 | 58.82 | 105.84 | 3.4014 |
| 84 | V.24 | 15 30 32 36 | -26.9 | 58.13 | 105.82 | 3.4020 |
| 99 | V.25 | 16 15 16 20 | -25.9 | 57.81 | 105.81 | 3.4023 |
| 126 | V.27 | 14 02 04 10 | -23.9 | 57.18 | 105.78 | 3.4033 |

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## TABLE 11 d.c.

| Nr. of Obs. | Nx / sin Nx | 2Nx / sin 2Nx | -b sin Nx / b + 12.83425 | -c sin 2Nx / c = 0.898398 | $Nx_o$ | $2Nx_o$ | $T_n - T_p$ |
|---|---|---|---|---|---|---|---|
| | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 53 | -58°48' | -117°36' | | | | | |
| | -0.85536 | -0.88620 | +10.979 | +0.796 | -47°02' | -94°04' | |
| | -0.73175 | -0.99748 | + 9.391 | +0.896 | -48 31 | -97 02 | |
| 54 | -0.74915 | -0.99248 | + 9.614 | +0.892 | -48 17 | -96 34 | |
| | -0.74644 | -0.99344 | + 9.580 | +0.893 | -48 20 | -96 40 | $-14^m.21$ |
| | -0.74703 | -0.99324 | + 9.587 | +0.892 | -48 19 | | |
| 84 | -58°12' | -116°24' | | | | | |
| | -0.84989 | -0.89571 | +10.908 | +0.805 | -46 29 | -92 58 | |
| | -0.72517 | -0.99866 | + 9.307 | +0.897 | -48 00 | -96 00 | |
| | -0.74314 | -0.99452 | + 9.538 | +0.893 | -47 43 | | -14.04 |
| 99 | -57°48' | -115°36' | | | | | |
| | -0.84619 | -0.90183 | +10.860 | +0.810 | -46 08 | -92 16 | |
| | -0.72095 | -0.99922 | + 9.255 | +0.898 | -47 39 | -95 18 | |
| | -0.73904 | -0.99572 | + 9.485 | +0.895 | -47 27 | | -13.95 |
| 126 | -57°12' | -114°24' | | | | | |
| | -0.84057 | -0.91068 | +10.788 | +0.818 | -45 35 | -91 10 | |
| | -0.71427 | -0.99979 | + 9.167 | +0.898 | -47 08 | -94 16 | |
| | -0.73294 | -0.99723 | + 9.407 | +0.896 | -46 54 | -93 48 | |
| | -0.73016 | -0.99780 | + 9.371 | +0.896 | -46 56 | | -13.80 |

## TABLE 11 d.c.

| Nr. of obs. | P | $(n \cdot P)^m$ | $(n \cdot P)^d$ | $T_i$ |
|---|---|---|---|---|
| | 14 | 15 | 16 | 17 |
| 53 | $105^m.7932$ | $13964^m.7024$ | $-9^d16^h44^m.70$ | $8^h24^m.80$ |
| 54 | 105.7932 | 13858.9092 | -9 14 58.91 | 10 10.59 |
| 84 | 105.7822 | 10684.0022 | -7 10 04.22 | 15 05.28 |
| 99 | 105.7768 | 9202.7768 | -6 09 22.58 | 15 46.92 |
| 126 | 105.7658 | 6451.7138 | -4 11 31.71 | 13 37.79 |

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## TABLE 11 d.c.

| Nr. of obs. | $T_i^d$ | $T_i - T_{VI}1^d_0$ | $\alpha^o$ | $a^h$ | $T_i - T_o^d$ $T_o = VI.1^d.0482$ | No. of complete revolutions n |
|---|---|---|---|---|---|---|
| | 18 | 19 | 20 | 21 | 22 | 23 |
| 53 | V.22$^d$.3583 <br> 3597 <br> 3604 | -9$^d$.6417 | 357$^o$.916 <br> ÷ 38 | 23$^h$51$^m$7 <br> ÷ 2.3 | -9.6899 | -132 |
| 54 | V.22.4312 <br> 4340 <br> 4354 | -9.5688 | 357.729 <br> ÷ 36 | 23 50.9 <br> ÷ 2.2 | -9.6170 | -131 |
| 55 | V.24.6458 <br> 6472 <br> 6500 | -7.3542 | 351.971 <br> ÷ 49 | 23 27.9 <br> ÷ 2.9 | -7.4024 | -101 |
| 99 | V.25.6771 <br> 6778 <br> 6806 | -6.3229 | 349.290 <br> ÷ 50 | 23 17.2 <br> ÷ 3.0 | -6.3711 | - 87 |
| 126 | V.27.5847 <br> 5861 <br> 5903 | -4.4153 | 344.330 <br> ÷ 36 | 22 57.3 <br> ÷ 2.2 | -4.4635 | - 61 |

## TABLE 11 d.c.

| Nr. of obs. | $n\Delta\lambda_o$ | $n^2\delta\lambda_o$ | $\lambda^o_{oi}$ | $\lambda^h_{oi}$ | $\Theta = \alpha - \lambda_{oi}$ |
|---|---|---|---|---|---|
| | 24 | 25 | 26 | 27 | 28 |
| 53 | ÷3526$^o$.776 | +4$^o$.008 | 355$^o$.060 | 23$^h$40$^m$3 | ÷0$^h$11$^m$4 |
| 54 | ÷3500.058 | +3.947 | 328.281 | 21 53.1 | +1 57.8 |
| 84 | ÷2698.518 | +2.346 | 245.140 | 16 20.6 | ÷7 07.3 |
| 99 | ÷2324.466 | +1.714 | 230.483 | 15 22.0 | ÷7 55.2 |
| 126 | ÷1629.798 | +0.856 | 254.930 | 16 59.7 | ÷5 57.6 |

## TABLE 11 d.c.

| Nr. of obs. | $\lambda d$ | $T_{\Omega}$ | $\dfrac{(30) \div (17)}{2}$ | $T_p$ |
|---|---|---|---|---|
| | 29 | 30 | 31 | 32 |
| 53 | -0.0146 | 8$^h$15$^m$8 | 8$^h$20$^m$3 | 8$^h$34$^m$5 |
| 54 | -0.0139 | 10 02.0 | 10 06.3 | 10 20.5 |
| 84 | -0.0188 | 15 03.4 | 15 04.3 | 15 18.3 |
| 99 | -0.0194 | 15 47.3 | 15 47.1 | 16 01.1 |
| 126 | -0.0139 | 13 41.4 | 13 39.6 | 15 53.4 |

4. Having orbit thus prepared we are able to proceed to the computation of the final coordinates of the subsatellite points. We shall again use formulas (21), (22), (43), and (45), for the computation of $\varphi_s$ and $\lambda_s$. Value u appearing in them we obtain from transformed formula (18):

$$u = \omega + M + 2e \sin M + \frac{5}{4} e^2 \sin 2M,$$

where

$$M = (T_i - T_p) \frac{360}{P}. \tag{15}$$

Table 12 contains computations made with the above formulas. Here is the content of the separate columns, in numerical order:

Col. 1 and 2 - date and moment of observation $T_i$;
Col. 3 - $(T_i - T_p)$;
Col. 4 - M according to formula (48);
Col. 5 - 10 - computation of u according to formula (47);
Col. 11 - 13 - computation $\varphi_s$, formula (21);
Col. 14 - 16 - computation $\Delta\lambda'$ agreeing with formula (22);
Col. 17 - 19 - computation of $\Delta\lambda''$ according to formula:

$$\Delta\lambda'' = \frac{T_i - T}{P} \cdot \Delta\lambda_0 \; ;$$

Col. 20 - $\lambda_{0i}$ from table 1, col. 26;
Col. 21 - $\lambda_s = (20) + (16) + (19)$.

TABLE 12

| Nr. of obs. | Date | $T_i$ | $T_i - T_p$ | M | 2M | sin M |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 |
| 53 | V.22$^d$ | $8^h 36^m$ | + 1.5 | 5°102 | 10°12'2 | +0.0889 |
| | | 38 | 3.5 | 11.905 | 23 38.6 | +0.2062 |
| | | 39 | 4.5 | 15.306 | 30 36.8 | 0.2639 |
| 54 | " | 10 21 | + 0.5 | 1.701 | 3 24.2 | +0.0297 |
| | | 25 | 4.5 | 15.306 | 30 36.8 | 0.2639 |
| | | 27 | 6.5 | 22.109 | 44 13.0 | 0.3762 |
| 84 | V. 24 | 15 30 | +11.7 | 39.803 | 79 36.4 | +0.6401 |
| | | 32 | 13.7 | 46.607 | 93 12.8 | 0.7266 |
| | | 36 | 17.7 | 60.215 | 120 25.8 | 0.8679 |
| 99 | V. 25 | 16 15 | +13.9 | 47.292 | 94 35.0 | +0.7349 |
| | | 16 | 14.9 | 50.694 | 101 23.2 | 0.7738 |
| | | 20 | 18.9 | 64.303 | 128 36.4 | 0.9011 |
| 126 | V. 27 | 14 02 | + 8.6 | 29.268 | 58 32.2 | +0.4889 |
| | | 04 | 10.6 | 36.075 | 72 09.0 | 0.5887 |
| | | 10 | 16.6 | 56.495 | 112 59.4 | 0.9206 |

## TABLE 12 d.c.

| Nr. of obs. | sin 2M | 2e sin M | $\frac{5}{4} e^2 \sin 2M$ | u | sin u | sin g | $\varphi_s$ |
|---|---|---|---|---|---|---|---|
| | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 53 | ÷0.1771 | ÷ 1.141 | ÷0.159 | ÷ 65°.262 | ÷0.90826 | ÷0.82317 | ÷55°24' |
| | 0.4038 | 2.646 | 0.363 | 73.774 | 0.96013 | 0.87018 | 60 29 |
| | 0.5093 | 3.387 | 0.458 | 78.011 | 0.97821 | 0.88656 | 62 27 |
| 54 | ÷0.0593 | ÷ 0.381 | ÷0.053 | ÷ 60.955 | ÷0.87420 | ÷0.79230 | ÷52 24 |
| | 0.5093 | 3.387 | 0.458 | 77.971 | 0.97803 | 0.88640 | 62 26 |
| | 0.6974 | 4.828 | 0.627 | 86.384 | 0.99801 | 0.90451 | 64 45 |
| 55 | ÷0.9836 | ÷ 8.215 | ÷0.884 | ÷107.032 | ÷0.95613 | ÷0.86655 | ÷60 04 |
| | 0.9984 | 9.325 | 0.897 | 114.595 | 0.90655 | 0.82162 | 55 15 |
| | 0.8622 | 11.139 | 0.775 | 130.259 | 0.76304 | 0.69155 | 43 45 |
| 99 | ÷0.9968 | ÷ 9.432 | ÷0.896 | +115.432 | ÷0.90309 | ÷0.81848 | ÷54 56 |
| | 0.9803 | 9 .931 | 0.881 | 119.316 | 0.87193 | 0.79024 | 52 12 |
| | 0.7815 | 11.565 | 0.702 | 134.380 | 0.71448 | 0.64772 | 40 22 |
| 126 | ÷0.8529 | ÷ 6.275 | ÷0.766 | ÷ 93.489 | ÷0.99815 | ÷0.90463 | ÷64 46 |
| | 0.9519 | 7.556 | 0.855 | 101.666 | 0.97934 | 0.88759 | 62 34 |
| | 0.9206 | 10.702 | 0.827 | 125.204 | 0.81714 | 0.74058 | 47 47 |

$$\sin i = 0.90631$$

## TABLE 12 d.c.

| Nr.of obs. | tg u | tg Δλ | Δλ | $T_i - T_{\text{☊}}$ | $\dfrac{T_i - T_{\text{☊}}}{p}$ | δλ | $\lambda_{oi}$ | $\lambda_s$ |
|---|---|---|---|---|---|---|---|---|
| | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
| 53 | ÷ 2.17083 | ÷0.91744 | 42°32' | 15.7 | 0.1483 | -3°58' | 355°04 | 33°38 |
| | ÷ 3.4346 | 1.45153 | 55 20 | 17.7 | 0.1672 | 4 28 | | 46 02 |
| | ÷ 4.7114 | 1.99113 | 63 20 | 17.7 | 0.1767 | 4 43 | | 53 41 |
| 54 | ÷ 1.80034 | ÷0.76085 | 37 18 | 17.7 | 0.1389 | -3 43 | 328 17 | 1 50 |
| | ÷ 4.6912 | +1.98259 | 63 14 | 17.7 | 0.1767 | 4 43 | | 26 48 |
| | ÷15.821 | ÷6.6863 | 81 30 | 20.7 | 0.1955 | 5 14 | | 44 33 |
| 84 | - 3.2641 | -1.37947 | 125 56 | 25.7 | 0.2428 | -6 30 | 245 08 | 4 34 |
| | - 2.14777 | -0.90769 | 137 46 | 27.7 | 0.2617 | 7 00 | | 15 54 |
| | - 1.18055 | -0.49892 | 153 29 | 31.7 | 0.2996 | 8 01 | | 30 36 |
| 99 | - 2.10284 | -0.88870 | 138 22 | 27.9 | 0.2637 | -7 03 | 230 29 | 1 48 |
| | - 1.78077 | -0.75259 | 143 02 | 28.9 | 0.2731 | 7 18 | | 6 13 |
| | - 1.02176 | -0.43182 | 156 39 | 32.9 | 0.3109 | 8 19 | | 18 49 |
| 126 | -16.428 | -6.9428 | 98 12 | 22.4 | 0.2117 | -5 40 | 254 56 | 347 28 |
| | - 4.8430 | -2.04675 | 116 02 | 24.4 | 0.2306 | 6 10 | | 4 48 |
| | - 1.41759 | -0.59910 | 149 04 | 30.4 | 0.2874 | 7 41 | | 36 19 |

$$\cos i = 0.42262$$

Still remaining to be computed: satellite distance from the
place of observation - l, and its height above the earth's surface - H.
The first of these values we compute with the formula:

$$\cos l = \sin \varphi_z \sin \varphi_s + \cos \varphi_z \cos \varphi_s \cos \Delta \lambda$$

## TABLE 13

| Nr. of obs. | $\sin \varphi_s$ | $\cos \varphi_s$ | $\sin \varphi_s \sin \varphi_z$ | $\cos \varphi_s \cos \varphi_z$ | $\Delta \lambda$ | $\cos \Delta \lambda$ | $\cos l$ |
|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| 53 | 0.823 14 | 0.567 84 | 0.652 66 | 0.346 02 | 12°36' | 0.975 92 | 0.990 35 |
| | 0.870 21 | 0.492 68 | 0.689 98 | 0.300 22 | 25 00 | 0.906 31 | 0.962 07 |
| | 0.886 61 | 0.462 52 | 0.702 98 | 0.281 84 | 32 39 | 0.841 98 | 0.940 28 |
| 54 | 0.792 29 | 0.610 15 | 0.628 20 | 0.371 80 | 19 12 | 0.944 38 | 0.979 32 |
| | 0.886 47 | 0.462 78 | 0.702 87 | 0.282 00 | 5 46 | 0.994 94 | 0.983 44 |
| | 0.904 46 | 0.426 57 | 0.717 14 | 0.259 93 | 23 31 | 0.916 94 | 0.955 48 |
| 84 | 0.866 61 | 0.498 99 | 0.687 13 | 0.304 06 | 16 28 | 0.958 98 | 0.978 72 |
| | 0.821 65 | 0.570 00 | 0.651 48 | 0.347 34 | 5 08 | 0.995 99 | 0.997 43 |
| | 0.691 51 | 0.722 36 | 0.548 29 | 0.440 18 | 9 34 | 0.986 09 | 0.982 35 |
| 99 | 0.818 48 | 0.574 53 | 0.648 96 | 0.350 10 | 19 14 | 0.944 18 | 0.979 52 |
| | 0.790 16 | 0.612 91 | 0.626 51 | 9.373 48 | 14 49 | 0.966 75 | 0.987 57 |
| | 0.647 68 | 0.761 92 | 0.513 54 | 0.464 28 | 2 13 | 0.999 25 | 0.977 47 |
| 126 | 0.904 58 | 0.426 31 | 0.717 23 | 0.259 78 | 32 34 | 0.842 77 | 0.936 16 |
| | 0.887 55 | 0.460 72 | 0.703 73 | 0.280 74 | 16 14 | 0.960 13 | 0.973 28 |
| | 0.740 61 | 0.671 94 | 0.587 22 | 0.409 45 | 15 17 | 0.964 63 | 0.982 19 |

$$\sin \varphi_z = 0.792\ 89 \qquad \cos \varphi_z = 0.609\ 36$$

## TABLE 13 d.c.

| Nr. of obs. | $l^\circ$ | l km | cos M | cos 2M | -0.006 272 (cos 2M - 1) | -0.112 cosM | r | H |
|---|---|---|---|---|---|---|---|---|
| | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 53 | 7°58' | 887 | +0.996 0 | +0.984 2 | +0.000 10 | -0.111 55 | 6578 | 207 |
| | 15 50 | 1762 | 0.978 5 | +0.914 8 | +0.000 53 | -0.109 59 | 6596 | 225 |
| | 19 54 | 2215 | 0.964 6 | +0.860 6 | +0.000 87 | -0.108 04 | 6610 | 239 |
| 54 | 11 40 | 1299 | +0.999 6 | +0.998 2 | +0.000 01 | -0.111 96 | 6574 | 203 |
| | 10 26 | 1161 | 0.964 6 | +0.860 6 | +0.000 87 | -0.108 04 | 6610 | 239 |
| | 17 10 | 1911 | 0.926 5 | +0.716 7 | +0.001 78 | -0.103 77 | 6648 | 277 |
| 84 | 11 50 | 1317 | +0.768 3 | +0.180 5 | +0.005 14 | -0.086 05 | 6804 | 433 |
| | 4 07 | 458 | 0.687 1 | -0.056 1 | +0.006 62 | -0.076 96 | 6882 | 511 |
| | 10 47 | 1200 | 0.496 7 | -0.506 5 | +0.009 45 | -0.055 63 | 7061 | 690 |
| 99 | 11 37 | 1293 | +0.678 2 | -0.079 9 | +0.006 77 | -0.075 96 | 6891 | 5 20 |
| | 9 03 | 1007 | 0.633 4 | -0.194 0 | +0.007 49 | -0.070 94 | 6933 | 562 |
| | 12 11 | 1356 | 0.433 7 | -0.523 9 | +0.010 19 | -0.048 57 | 7119 | 748 |
| 126 | 20 35 | 2291 | +0.872 4 | +0.522 0 | +0.003 00 | -0.097 71 | 6702 | 331 |
| | 13 16 | 477 | 0.808 3 | +0.306 5 | +0.004 35 | -0.090 53 | 6765 | 394 |
| | 10 50 | 1206 | 0.551 9 | -0.390 5 | +0.008 72 | -0.061 81 | 7010 | 639 |

while the second is obtained:

$$H = r - R = 7403 [1 - 0.112 \cos M - 0.006272(\cos 2M - 1)] - 6371.$$

These calculations are given in Table 13.

## I. Final Considerations

    Under the conditions described, with only a partial knowledge of the orbit elements, the way of solving the problem presented seems the most reasonable. Attempts at computation of the missing orbit elements were also made with another method, on the basis of selected observations (Polish, Soviet, Czechoslovak, German) made on the local meridian or first vertical. Considering, however, the necessity of extrapolation and the insufficiently accurate execution of the above condition, suitable results were taken only in approximation. For several months from the time the problem was completed, we had no accurate data relative to Sputnik III orbit elements that would facilitate a check and comparison of the values obtained. Only immediately before giving this work to the publisher did we receive a publication from the German Federal Republic, in which values were listed which had been computed on the basis of sufficiently rich initial data and which indicated agreement with those found by us. (17)

    Thus, the value for the change in the daily right ascension of the ascending node $\Delta \Lambda_{\Omega} 1/d = -2°55$ given in (17) is included in the realm of values computed in the framework of our treatment, with the theoretical formula - $-2°51$ and empirically $-2°63$. And then the moment found by us for the first satellite passage through the ascending node on 1 June 1958, which we chose as an initial epoch, is identical with (17) $T_{\Omega} = VI.1^d 1^h 09^m 28^s = VI.1.^d 0.48\ 24$. In a similar way, comparison of the change of the daily geographic longitude of the ascending node, caused by the motion of the earth's rotation yielded: (17) $\ldots \Delta \Lambda_{\Omega} = -26°76$, ours $\ldots \Delta \Lambda_{\Omega} = -26.72$. Nor did the other elements indicate any discrepancies impairing the accuracy of the determinations.

    On the basis of the fragments of our treatment above and the tables of results, one can see the accuracy we were able to achieve. It is satisfactory as far as the practical goal, dictated by the conditions named in paragraphs 6 and 7, is concerned; but the obtained data, computed with approximation formulas, relying on an insufficient knowledge of the orbit elements and a small number of astronomical observations, can only be considered approximate results. For example, to compute accurately the orbit elements of the first Soviet sputnik, the Moscow Institute of Theoretical Astronomy used 60,000 radio observations and 400 visual observations; for the second sputnik 13,000 radio and 2,000 visual observations were used; and, finally, in the case of the third Soviet satellite, merely in regard to the epoch, in the six weeks following its launching 53,000 radio and 1,260 optical observations were processed. All computations were made on high-speed electronic computers.

Having the above comparison in mind, we wish to emphasize the experimental nature of our treatment.

Finally, we wish to carry out a pleasant obligation by thanking Docent Doctor W. Opalski for valuable comments during the execution of the work and for checking the manuscript.

## FIGURE CAPTIONS

Center of Orbit          Center of the earth

Diagram 1

Greenwich Meridian

Equator

Diagram 2

Satellite orbit

Diagram 3

Diagram 4

Diagram 5

Diagram 6

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